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GRADE 3 • MODULE 1

Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

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OVERVIEW

This 25-day module begins the year by building on students' fluency with addition and their knowledge of arrays. In Topic A, students initially use repeated addition to find the total from a number of equal groups (**2.OA.4**). As students notice patterns, they let go of longer addition sentences in favor of more efficient multiplication facts (**3.OA.1**). Lessons in Topic A move students' Grade 2 work with arrays and repeated addition a step further by developing skip-counting rows as a strategy for multiplication. Arrays become a cornerstone of the module. Students use the language of multiplication as they understand what factors are and differentiate between the size of groups and the number of groups within a given context. In this module, the factors 2, 3, 4, 5, and 10 provide an entry point for moving into more difficult factors in later modules.

The study of factors links Topics A and B; Topic B extends the study to division. Students understand division as an unknown factor problem and relate the meaning of unknown factors to either the number or the size of groups (**3.OA.2**, **3.OA.6**). By the end of Topic B, students are aware of a fundamental connection between multiplication and division that lays the foundation for the rest of the module.

In Topic C, students use the array model and familiar skip-counting strategies to solidify their understanding of multiplication and practice related facts of 2 and 3. They become fluent enough with arithmetic patterns to *add* or *subtract* groups from known products to solve more complex multiplication problems (**3.OA.1**). They apply their skills to word problems using drawings and equations with a symbol to find the unknown factor (**3.OA.3**). This culminates in students using arrays to model the distributive property as they decompose units to multiply (**3.OA.5**).

The Distributive Property
$6 \times 4 = \underline{\hspace{1cm}}$

$(5 \times 4) = 20$

$(1 \times 4) = 4$

$(6 \times 4) = (5 \times 4) + (1 \times 4)$
$= 20 + 4$

In Topic D, students model, write, and solve partitive and measurement division problems with 2 and 3 (**3.OA.2**). Consistent skip-counting strategies and the continued use of array models are pathways for students to naturally relate multiplication and division. Modeling advances as students use tape diagrams to represent multiplication and division. A final lesson in this topic solidifies a growing understanding of the relationship between operations (**3.OA.7**).

Topic E shifts students from simple understanding to analyzing the relationship between multiplication and division. Practice of both operations is combined—this time using units of 4—and a lesson is explicitly dedicated to modeling the connection between them (**3.OA.7**). Skip-counting, the distributive property, arrays, number bonds, and tape diagrams are tools for both operations (**3.OA.1**, **3.OA.2**). A final lesson invites students to explore their work with arrays and related facts through the lens of the commutative property as it relates to multiplication (**3.OA.5**).

The Commutative Property

3 rows of 5   5 rows of 3

$3 \times 5 = 5 \times 3$

Topic F introduces the factors 5 and 10, familiar from skip-counting in Grade 2. Students apply the multiplication and division strategies they have used to mixed practice with all of the factors included in Module 1 (**3.OA.1, 3.OA.2, 3.OA.3**). Students model relationships between factors, analyzing the arithmetic patterns that emerge to compose and decompose numbers, as they further explore the relationship between multiplication and division (**3.OA.3**, **3.OA.5**, **3.OA.7**).

In the final lesson of the module, students apply the tools, representations, and concepts they have learned to problem solving with multi-step word problems using all four operations (**3.OA.3, 3.OA.8**). They demonstrate the flexibility of their thinking as they assess the reasonableness of their answers for a variety of problem types.
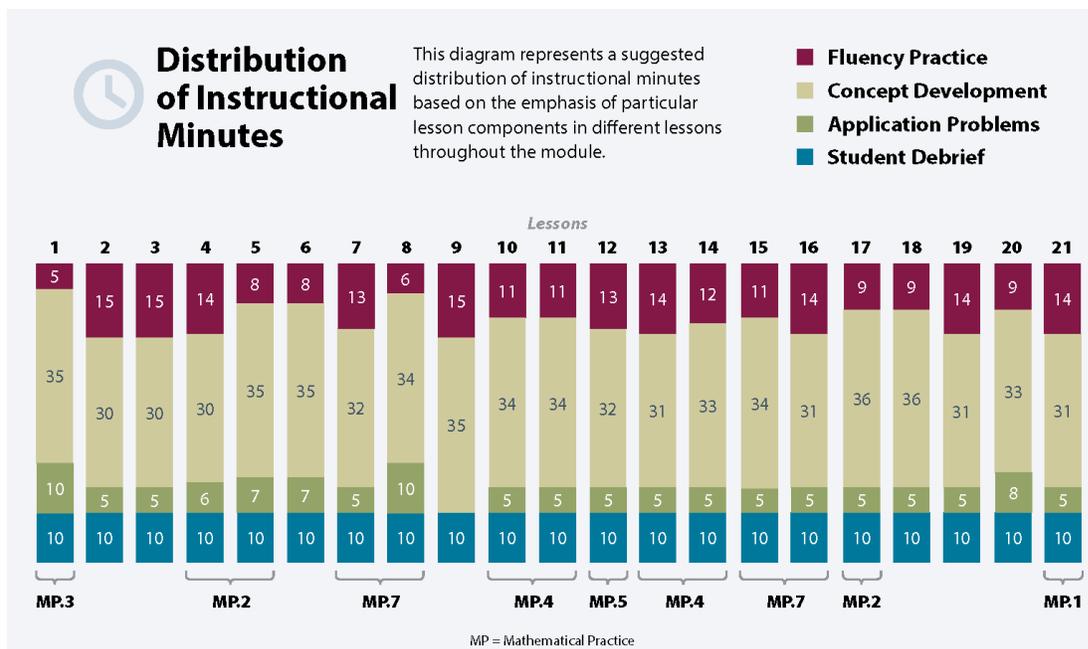
The Mid-Module Assessment follows Topic C. The End-of-Module Assessment follows Topic F.

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Consolidate Lessons 12 and 13, both of which are division lessons sharing the same objective. Include units of 2 and units of 3 in the consolidated lesson.

Omit Lessons 15 and 19. Lesson 15 uses the tape diagram to provide a new perspective on the commutative property, a concept students have studied since Lesson 7. Lesson 19 introduces the significant complexity of the distributive property with division. The concepts from both lessons are reinforced within Module 3.

Distribution of Instructional Minutes

This diagram represents a suggested distribution of instructional minutes based on the emphasis of particular lesson components in different lessons throughout the module.

■ Fluency Practice
■ Concept Development
■ Application Problems
■ Student Debrief

MP = Mathematical Practice

Module 1:   Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

Focus Grade Level Standards

Represent and solve problems involving multiplication and division.[1]

3.OA.1   Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

3.OA.2   Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.3   Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Glossary, Table 2.)

3.OA.4   Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*

Understand properties of multiplication and the relationship between multiplication and division.[2]

3.OA.5   Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*[3]

3.OA.6   Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.[4]

3.OA.7   Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

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[1]Limited to factors of 2–5 and 10 and the corresponding dividends in this module.
[2]Limited to factors of 2–5 and 10 and the corresponding dividends in this module.
[3]The associative property is addressed in Module 3.
[4]Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.[5]

3.OA.8 — Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

Foundational Standards

2.OA.3 — Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

2.OA.4 — Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

2.NBT.2 — Count within 1000; skip-count by 5s, 10s, and 100s.

Focus Standards for Mathematical Practice

MP.1 — **Make sense of problems and persevere in solving them.** Students model multiplication and division using the array model. They solve two-step mixed word problems and assess the reasonableness of their solutions.

MP.2 — **Reason abstractly and quantitatively.** Students make sense of quantities and their relationships as they explore the properties of multiplication and division and the relationship between them. Students decontextualize when representing equal group situations as multiplication and when they represent division as partitioning objects into equal shares or as unknown factor problems. Students contextualize when they consider the value of units and understand the meaning of the quantities as they compute.

MP.3 — **Construct viable arguments and critique the reasoning of others.** Students represent and solve multiplication and division problems using arrays and equations. As they compare methods, they construct arguments and critique the reasoning of others. This practice is particularly exemplified in daily Application Problems and in specific lessons dedicated to problem solving in which students solve and reason with others about their work.

MP.4 — **Model with mathematics.** Students represent equal groups using arrays and equations to multiply, divide, add, and subtract.

MP.7 — **Look for and make use of structure.** Students notice structure when they represent quantities by using drawings and equations to represent the commutative and distributive properties. The relationship between multiplication and division also highlights structure for students as they determine the unknown whole number in a multiplication or division equation.

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[5]In this module, problem solving is limited to factors of 2–5 and 10 and the corresponding dividends. 3.OA.9 is addressed in Module 3.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
3.OA.1 3.OA.3	A	**Multiplication and the Meaning of the Factors** Lesson 1: Understand *equal groups of* as multiplication. Lesson 2: Relate multiplication to the array model. Lesson 3: Interpret the meaning of factors—the size of the group or the number of groups.	3
3.OA.2 **3.OA.6** 3.OA.3 3.OA.4	B	**Division as an Unknown Factor Problem** Lesson 4: Understand the meaning of the unknown as the size of the group in division. Lesson 5: Understand the meaning of the unknown as the number of groups in division. Lesson 6: Interpret the unknown in division using the array model.	3
3.OA.1 **3.OA.5** 3.OA.3 3.OA.4	C	**Multiplication Using Units of 2 and 3** Lessons 7–8: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models. Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models. Lesson 10: Model the distributive property with arrays to decompose units as a strategy to multiply.	4
		Mid-Module Assessment: Topics A–C (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
3.OA.2 **3.OA.4** **3.OA.6** **3.OA.7** 3.OA.3 3.OA.8	D	**Division Using Units of 2 and 3** Lesson 11: Model division as the unknown factor in multiplication using arrays and tape diagrams. Lesson 12: Interpret the quotient as the number of groups or the number of objects in each group using units of 2. Lesson 13: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.	3

Standards		Topics and Objectives	Days
3.OA.5 **3.OA.7** 3.OA.1 3.OA.2 3.OA.3 3.OA.4 3.OA.6	E	**Multiplication and Division Using Units of 4** Lesson 14: Skip-count objects in models to build fluency with multiplication facts using units of 4. Lesson 15: Relate arrays to tape diagrams to model the commutative property of multiplication. Lesson 16: Use the distributive property as a strategy to find related multiplication facts. Lesson 17: Model the relationship between multiplication and division.	4
3.OA.3 **3.OA.5** **3.OA.7** **3.OA.8** 3.OA.1 3.OA.2 3.OA.4 3.OA.6	F	**Distributive Property and Problem Solving Using Units of 2–5 and 10** Lessons 18–19: Apply the distributive property to decompose units. Lesson 20: Solve two-step word problems involving multiplication and division, and assess the reasonableness of answers. Lesson 21: Solve two-step word problems involving all four operations, and assess the reasonableness of answers.	4
		End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further application 1 day)	2
Total Number of Instructional Days			**25**

Terminology

New or Recently Introduced Terms

- Array[6] (arrangement of objects in rows and columns)
- Commutative property/commutative (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places)
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups)
- Distribute (with reference to the distributive property, e.g., in $12 \times 3 = (10 \times 3) + (2 \times 3)$ the 3 is the multiplier for each part of the decomposition)
- Divide/division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., $15 \div 5 = 3$)

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[6]Originally introduced in Grade 2, Module 6 but treated as new vocabulary in this module.

- Factors (numbers that are multiplied to obtain a product)
- Multiplication/multiply (an operation showing how many times a number is added to itself, e.g., 5 × 3 =15)
- Number of groups (factor in a multiplication problem that refers to the total equal groups)
- Parentheses (symbols ( ) used around an expression or numbers within an equation)
- Product (the answer when one number is multiplied by another)
- Quotient (the answer when one number is divided by another)
- Rotate (turn, used with reference to turning arrays 90 degrees)
- Row/column[7] (in reference to rectangular arrays)
- Size of groups (factor in a multiplication problem that refers to how many in a group)
- Unit (one segment of a partitioned tape diagram)
- Unknown (the missing factor or quantity in multiplication or division)

Familiar Terms and Symbols[8]

- Add 1 unit, subtract 1 unit (add or subtract a single unit of two, ten, etc.)
- Expression (see expanded description in box above)
- Number bond (illustrates part–part–whole relationship, shown at right)
- Ones, twos, threes, etc. (units of one, two, or three)
- Repeated addition (adding equal groups together, e.g., 2 + 2 + 2 + 2)
- Tape diagram (a method for modeling problems)
- Value (how much)

Suggested Tools and Representations

- 18 counters per student
- Tape diagram (a method for modeling problems)
- Number bond (shown at right)
- Array (arrangement of objects in rows and columns)

> **NOTES ON**
> **EXPRESSION, EQUATION, AND NUMBER SENTENCE:**
>
> Please note the descriptions for the following terms, which are frequently misused.
>
> - **Expression:** A number, or any combination of sums, differences, products, or divisions of numbers that evaluates to a number (e.g., 3 + 4, 8 × 3, 15 ÷ 3 as distinct from an equation or number sentence).
> - **Equation:** A statement that two expressions are equal (e.g., 3 × ___ = 12, 5 × b = 20, 3 + 2 = 5).
> - **Number sentence** (also addition, subtraction, multiplication, or division sentence): An equation or inequality for which both expressions are numerical and can be evaluated to a single number (e.g., 4 + 3 = 6 + 1, 2 = 2, 21 > 7 × 2, 5 ÷ 5 = 1). Number sentences are either true or false (e.g., 4 + 4 < 6 × 2 and 21 ÷ 7 = 4) and contain no unknowns.

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[7]Originally introduced in Grade 2, Module 6 but treated as new vocabulary in this module.
[8]These are terms and symbols students have used or seen previously.
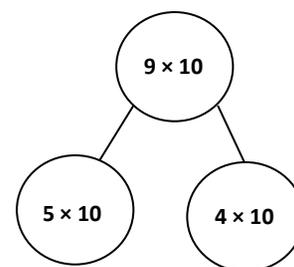
Suggested Methods of Instructional Delivery

Directions for Administration of Sprints

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration, teachers assume the role of athletic coaches. A rousing routine fuels students' motivation to do their personal best. Student recognition of increasing success is critical, and so every improvement is celebrated.

One Sprint has two parts with closely related problems on each. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more.

With practice, the following routine takes about 9 minutes.

Sprint A

Pass Sprint A out quickly, face down on student desks with instructions to not look at the problems until the signal is given. (Some Sprints include words. If necessary, prior to starting the Sprint, quickly review the words so that reading difficulty does not slow students down.)

T: You will have 60 seconds to do as many problems as you can. I do not expect you to finish all of them. Just do as many as you can, your personal best. (If some students are likely to finish before time is up, assign a number to count by on the back.)

T: Take your mark! Get set! THINK!

Students immediately turn papers over and work furiously to finish as many problems as they can in 60 seconds. Time precisely.

T: Stop! Circle the last problem you did. I will read just the answers. If you got it right, call out "Yes!" If you made a mistake, circle it. Ready?

T: (Energetically, rapid-fire call the first answer.)

S: Yes!

T: (Energetically, rapid-fire call the second answer.)

S: Yes!

Repeat to the end of Sprint A or until no student has a correct answer. If needed, read the count-by answers in the same way the Sprint answers were read. Each number counted-by on the back is considered a correct answer.

T: Fantastic! Now, write the number you got correct at the top of your page. This is your personal goal for Sprint B.

T: How many of you got one right? (All hands should go up.)

T: Keep your hand up until I say the number that is one more than the number you got correct. So, if you got 14 correct, when I say 15, your hand goes down. Ready?

T: (Continue quickly.) How many got two correct? Three? Four? Five? (Continue until all hands are down.)

If the class needs more practice with Sprint A, continue with the optional routine presented below.

T: I'll give you one minute to do more problems on this half of the Sprint. If you finish, stand behind your chair.

As students work, the student who scored highest on Sprint A might pass out Sprint B.

T: Stop! I will read just the answers. If you got it right, call out "Yes!" If you made a mistake, circle it. Ready? (Read the answers to the first half again as students stand.)

Movement

To keep the energy and fun going, always do a stretch or a movement game in between Sprints A and B. For example, the class might do jumping jacks while skip-counting by 5 for about 1 minute. Feeling invigorated, students take their seats for Sprint B, ready to make every effort to complete more problems this time.

Sprint B

Pass Sprint B out quickly, face down on student desks with instructions not to look at the problems until the signal is given. (Repeat the procedure for Sprint A up through the show of hands for how many right.)

T: Stand up if you got more correct on the second Sprint than on the first.

S: (Stand.)

T: Keep standing until I say the number that tells how many more you got right on Sprint B. If you got three more right on Sprint B than you did on Sprint A, when I say *three,* you sit down. Ready? (Call out numbers starting with one. Students sit as the number by which they improved is called. Celebrate the students who improved most with a cheer.)

T: Well done! Now, take a moment to go back and correct your mistakes. Think about what patterns you noticed in today's Sprint.

T: How did the patterns help you get better at solving the problems?

T: Rally Robin your thinking with your partner for 1 minute. Go!

Rally Robin is a style of sharing in which partners trade information back and forth, one statement at a time per person, for about 1 minute. This is an especially valuable part of the routine for students who benefit from their friends' support to identify patterns and try new strategies.

Students may take Sprints home.

RDW or Read, Draw, Write (an Equation and a Statement)

Mathematicians and teachers suggest a simple process applicable to all grades:

1. Read.
2. Draw and label.
3. Write an equation.
4. Write a word sentence (statement).

The more students participate in reasoning through problems with a systematic approach, the more they internalize those behaviors and thought processes.

- What do I see?
- Can I draw something?
- What conclusions can I make from my drawing?

Modeling with Interactive Questioning	Guided Practice	Independent Practice
The teacher models the whole process with interactive questioning, some choral response, and talk such as "What did Monique say, everyone?" After completing the problem, students might reflect with a partner on the steps they used to solve the problem. "Students, think back on what we did to solve this problem. What did we do first?" Students might then be given the same or a similar problem to solve for homework.	Each student has a copy of the question. Though guided by the teacher, they work independently at times and then come together again. Timing is important. Students might hear, "You have 2 minutes to do your drawing." Or, "Put your pencils down. Time to work together again." The Debrief might include selecting different student work to share.	Students are given a problem to solve and possibly a designated amount of time to solve it. The teacher circulates, supports, and thinks about which student work to show to support the mathematical objectives of the lesson. When sharing student work, students are encouraged to think about the work with questions such as, "What do you see that Jeremy did?" "What is the same about Jeremy's work and Sara's work?" "How did Jeremy show $\frac{3}{7}$ of the students?" "How did Sara show $\frac{3}{7}$ of the students?"

Personal White Boards

Materials Needed for Personal White Boards

1 heavy duty clear sheet protector
1 piece of stiff red tag board 11" × 8 ¼"
1 piece of stiff white tag board 11" × 8 ¼"
1 3" × 3" piece of dark synthetic cloth for an eraser (e.g., felt)
1 low odor blue dry erase marker, fine point

Directions for Creating Personal White Boards

Cut the white and red tag to specifications. Slide into the sheet protector. Store the eraser on the red side. Store markers in a separate container to avoid stretching the sheet protector.

Frequently Asked Questions About Personal White Boards

Why is one side red and one white?

- The white side of the board is the "paper." Students generally write on it, and if working individually, turn the board over to signal to the teacher they have completed their work. The teacher then says, "Show me your boards," when most of the class is ready.

What are some of the benefits of a personal white board?

- The teacher can respond quickly to gaps in student understandings and skills. "Let's do some of these on our personal white boards until we have more mastery."
- Students can erase quickly so that they do not have to suffer the evidence of their mistake.
- They are motivating. Students love both the drill and thrill capability and the chance to do story problems with an engaging medium.
- Checking work gives the teacher instant feedback about student understanding.

What is the benefit of this personal white board over a commercially purchased dry erase board?

- It is much less expensive.
- Templates such as place value charts, number bond mats, hundreds boards, and number lines can be stored between the two pieces of tag board for easy access and reuse.
- Worksheets, story problems, and other Problem Sets can be done without marking the paper so that students can work on the problems independently at another time.
- Strips with story problems, number lines, and arrays can be inserted and still have a full piece of paper on which to write.
- The red versus white side distinction clarifies expectations. When working collaboratively, there is no need to use the red side. When working independently, students know how to keep their work private.
- The tag board can be removed if necessary to project the work.

Scaffolds[9]

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*."

Preparing to Teach a Module

Preparation of lessons will be more effective and efficient if there has been an adequate analysis of the module first. Each module in *A Story of Units* can be compared to a chapter in a book. How is the module moving the plot, the mathematics, forward? What new learning is taking place? How are the topics and objectives building on one another? The following is a suggested process for preparing to teach a module.

Step 1: Get a preview of the plot.

A: Read the Table of Contents. At a high level, what is the plot of the module? How does the story develop across the topics?

B: Preview the module's Exit Tickets[10] to see the trajectory of the module's mathematics and the nature of the work students are expected to be able to do.

Note: When studying a PDF file, enter "Exit Ticket" into the search feature to navigate from one Exit Ticket to the next.

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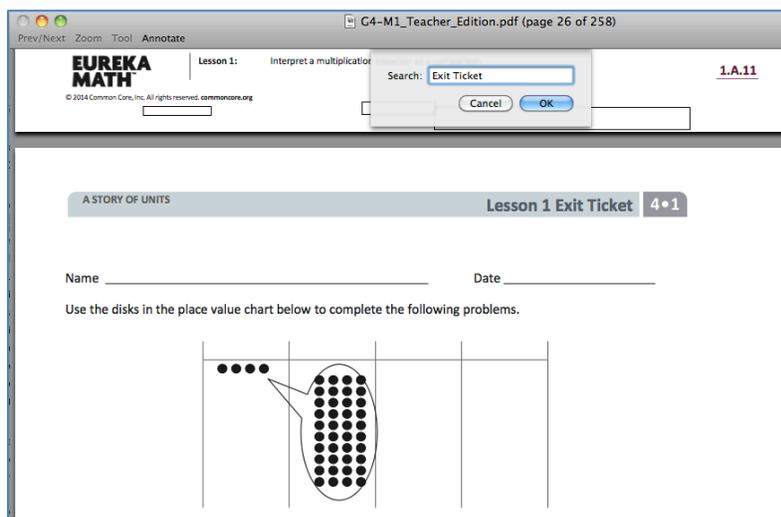
[9]Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website, www.p12.nysed.gov/specialed/aim, for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

[10] A more in-depth preview can be done by searching the Problem Sets rather than the Exit Tickets. Furthermore, this same process can be used to preview the coherence or flow of any component of the curriculum, such as Fluency Practice or Application Problems.

Step 2: Dig into the details.

A: Dig into a careful reading of the Module Overview. While reading the narrative, *liberally* reference the lessons and Topic Overviews to clarify the meaning of the text—the lessons demonstrate the strategies, show how to use the models, clarify vocabulary, and build understanding of concepts. Consider searching the video gallery on *Eureka Math*'s website to watch demonstrations of the use of models and other teaching techniques.

B: Having thoroughly investigated the Module Overview, read through the chart entitled Overview of Module Topics and Lesson Objectives to further discern the plot of the module. How do the topics flow and tell a coherent story? How do the objectives move from simple to complex?

Step 3: Summarize the story.

Complete the Mid- and End-of-Module Assessments. Use the strategies and models presented in the module to explain the thinking involved. Again, liberally reference the work done in the lessons to see how students who are learning with the curriculum might respond.

Preparing to Teach a Lesson

A three-step process is suggested to prepare a lesson. It is understood that at times teachers may need to make adjustments (customizations) to lessons to fit the time constraints and unique needs of their students. The recommended planning process is outlined below. Note: The ladder of Step 2 is a metaphor for the teaching sequence. The sequence can be seen not only at the macro level in the role that this lesson plays in the overall story, but also at the lesson level, where each rung in the ladder represents the next step in understanding or the next skill needed to reach the objective. To reach the objective, or the top of the ladder, all students must be able to access the first rung and each successive rung.

Step 1: Discern the plot.

A: Briefly review the module's Table of Contents, recalling the overall story of the module and analyzing the role of this lesson in the module.

B: Read the Topic Overview related to the lesson, and then review the Problem Set and Exit Ticket of each lesson in the topic.

C: Review the assessment following the topic, keeping in mind that assessments can be found midway through the module and at the end of the module.

Step 2: Find the ladder.

A: Complete the lesson's Problem Set.

B: Analyze and write notes on the new complexities of each problem as well as the sequences and progressions throughout problems (e.g., pictorial to abstract, smaller to larger numbers, single- to multi-step problems). The new complexities are the rungs of the ladder.

C: Anticipate where students might struggle, and write a note about the potential cause of the struggle.

D: Answer the Student Debrief questions, always anticipating how students will respond.
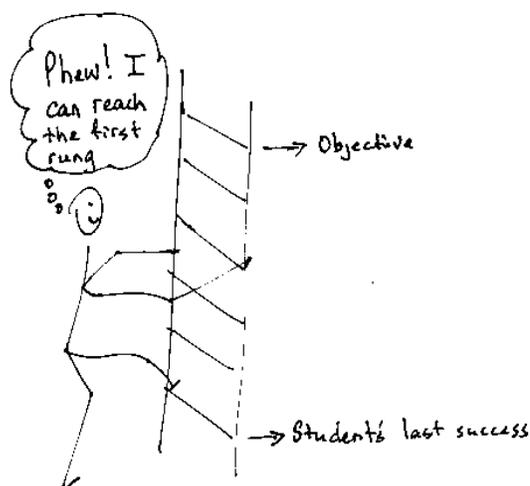
Step 3: Hone the lesson.

At times, the lesson and Problem Set are appropriate for all students and the day's schedule. At others, they may need customizing. If the decision is to customize based on either the needs of students or scheduling constraints, a suggestion is to decide upon and designate "Must Do" and "Could Do" problems.

A: Select "Must Do" problems from the Problem Set that meet the objective and provide a coherent experience for students; reference the ladder. The expectation is that the majority of the class will complete the "Must Do" problems within the allocated time. While choosing the "Must Do" problems, keep in mind the need for a balance of calculations, various word problem types[11], and work at both the pictorial and abstract levels.

B: "Must Do" problems might also include remedial work as necessary for the whole class, a small group, or individual students. Depending on anticipated difficulties, those problems might take different forms as shown in the chart below.

Anticipated Difficulty	"Must Do" Remedial Problem Suggestion
The first problem of the Problem Set is too challenging.	Write a short sequence of problems on the board that provides a ladder to Problem 1. Direct the class or small group to complete those first problems to empower them to begin the Problem Set. Consider labeling these problems "Zero Problems" since they are done prior to Problem 1.
There is too big of a jump in complexity between two problems.	Provide a problem or set of problems that creates a bridge between the two problems. Label them with the number of the problem they follow. For example, if the challenging jump is between Problems 2 and 3, consider labeling the bridging problems "Extra 2s."
Students lack fluency or foundational skills necessary for the lesson.	Before beginning the Problem Set, do a quick, engaging fluency exercise, such as a Rapid White Board Exchange, "Thrilling Drill," or Sprint. Before beginning any fluency activity for the first time, assess that students are poised for success with the easiest problem in the set.
More work is needed at the concrete or pictorial level.	Provide manipulatives or the opportunity to draw solution strategies. Especially in Kindergarten, at times the Problem Set or pencil and paper aspect might be completely excluded, allowing students to simply work with materials.
More work is needed at the abstract level.	Hone the Problem Set to reduce the amount of drawing as appropriate for certain students or the whole class.

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[11] See the Progression Documents "K, Counting and Cardinality" and "K–5, Operations and Algebraic Thinking" pp. 9 and 23, respectively.

C: "Could Do" problems are for students who work with greater fluency and understanding and can, therefore, complete more work within a given time frame. Adjust the Exit Ticket and Homework to reflect the "Must Do" problems or to address scheduling constraints.

D: At times, a particularly tricky problem might be designated as a "Challenge!" problem. This can be motivating, especially for advanced students. Consider creating the opportunity for students to share their "Challenge!" solutions with the class at a weekly session or on video.

E: Consider how to best use the vignettes of the Concept Development section of the lesson. Read through the vignettes, and highlight selected parts to be included in the delivery of instruction so that students can be independently successful on the assigned task.

F: Pay close attention to the questions chosen for the Student Debrief. Regularly ask students, "What was the lesson's learning goal today?" Help them articulate the goal.

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic C	Constructed response with rubric	3.OA.1 3.OA.2 3.OA.5 3.OA.6
End-of-Module Assessment Task	After Topic F	Constructed response with rubric	3.OA.1 3.OA.2 3.OA.3 3.OA.4 3.OA.5 3.OA.6 3.OA.7 3.OA.8