



## Table of Contents

# GRADE 3 • MODULE 3

## Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

<b>Module Overview</b> .....	2
Topic A: The Properties of Multiplication and Division.....	11
Topic B: Multiplication and Division Using Units of 6 and 7.....	48
Topic C: Multiplication and Division Using Units up to 8.....	94
<b>Mid-Module Assessment and Rubric</b> .....	140
Topic D: Multiplication and Division Using Units of 9.....	147
Topic E: Analysis of Patterns and Problem Solving Including Units of 0 and 1.....	197
Topic F: Multiplication of Single-Digit Factors and Multiples of 10.....	235
<b>End-of-Module Assessment and Rubric</b> .....	269
<b>Answer Key</b> .....	279

## Grade 3 • Module 3

# Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

## OVERVIEW

This 25-day module builds directly on students' work with multiplication and division in Module 1. At this point, Module 1 instruction coupled with fluency practice in Module 2 has students well on their way to meeting the Grade 3 fluency expectation for multiplying and dividing within 100 (**3.OA.7**). Module 3 extends the study of factors from 2, 3, 4, 5, and 10 to include all units from 0 to 10, as well as multiples of 10 within 100. Similar to the organization of Module 1, the introduction of new factors in Module 3 spreads across topics. This allows students to build fluency with facts involving a particular unit before moving on. The factors are sequenced to facilitate systematic instruction with increasingly sophisticated strategies and patterns.

Topic A begins by revisiting the commutative property. Students study familiar facts from Module 1 to identify known facts using units of 6, 7, 8, and 9 (**3.OA.5, 3.OA.7**). They realize that they already know more than half of their facts by recognizing, for example, that if they know  $2 \times 8$ , they also know  $8 \times 2$  through commutativity. This begins a study of arithmetic patterns that becomes an increasingly prominent theme in the module (**3.OA.9**). The subsequent lesson carries this study a step further; students apply the commutative property to relate  $5 \times 8$  and  $8 \times 5$  and then add one more group of 8 to solve  $6 \times 8$  and, by extension,  $8 \times 6$ . The final lesson in this topic builds fluency with familiar multiplication and division facts, preparing students for the work ahead by introducing the use of a letter to represent the unknown in various positions (**3.OA.3, 3.OA.4**).

Topic B introduces units of 6 and 7, factors that are well suited to Level 2 skip-counting strategies and to the Level 3 distributive property strategy, already familiar from Module 1. Students learn to compose up to and then over the next ten. For example, to solve a fact using units of 7, they might count 7, 14, and then mentally add  $14 + 6 + 1$  to make 21. This strategy previews the associative property using addition and illuminates patterns as students apply count-bys to solve problems. In the next lesson, students apply the distributive property (familiar from Module 1) as a strategy to multiply and divide. They decompose larger unknown facts into smaller known facts to solve. For example,  $48 \div 6$  becomes  $(30 \div 6) + (18 \div 6)$ , or  $5 + 3$  (**3.OA.5, 3.OA.7**). Topic B's final lesson emphasizes word problems, providing opportunities to analyze and model. Students apply the skill of using a letter to represent the unknown in various positions within multiplication and division problems (**3.OA.3, 3.OA.4, 3.OA.7**).

Topic C anticipates the formal introduction of the associative property with a lesson focused on making use of structure to problem solve. Students learn the conventional order for performing operations when parentheses are and are not present in an equation (**3.OA.8**). With this student knowledge in place, the associative property emerges in the next lessons as a strategy to multiply using units up to 8 (**3.OA.5**). Units of 6 and 8 are particularly useful for presenting this Level 3 strategy. Rewriting 6 as  $2 \times 3$  or 8 as  $2 \times 4$  makes shifts in grouping readily apparent (see example on next page) and also utilizes the familiar factors 2,

3, and 4 as students learn the new material. The following strategy may be used to solve a problem like  $8 \times 5$ :

$$8 \times 5 = (4 \times 2) \times 5$$

$$8 \times 5 = 4 \times (2 \times 5)$$

$$8 \times 5 = 4 \times 10$$

In the final lesson of Topic C, students relate division to multiplication using units up to 8. They understand division as both a quantity divided into equal groups and an unknown factor problem for which—given the large size of units—skip-counting to solve can be more efficient than dividing (**3.OA.3**, **3.OA.4**, **3.OA.7**).

Topic D introduces units of 9 over three days, with students exploring a variety of arithmetic patterns that become engaging strategies for quickly learning facts with automaticity (**3.OA.3**, **3.OA.7**, **3.OA.9**). Nines are placed late in the module so that students have enough experience with multiplication and division to recognize, analyze, and apply the rich patterns found in the manipulation of units of 9. As with other topics, the sequence ends with interpreting the unknown factor to solve multiplication and division problems (**3.OA.3**, **3.OA.4**, **3.OA.5**, **3.OA.7**).

In Topic E, students begin by working with facts using units of 0 and 1. From a procedural standpoint, these are simple facts that require little time for students to master; however, understanding the concept of nothing (zero) is more complex, particularly as it relates to division. This unique combination of simple and complex explains the late introduction of 0 and 1 in the sequence of factors. Students study the results of multiplying and dividing with units of 0 and 1 to identify relationships and patterns (**3.OA.7**, **3.OA.9**). The topic closes with a lesson devoted to two-step problems involving all four operations (**3.OA.8**). In this lesson, students work with equations involving unknown quantities and apply the rounding skills learned in Module 2 to make estimations that help them assess the reasonableness of their solutions (**3.OA.8**).

In Topic F, students multiply by multiples of 10 (**3.NBT.3**). To solve a fact like  $2 \times 30$ , they first model the basic fact  $2 \times 3$  on the place value chart. Place value understanding helps them to notice that the product shifts one place value to the left when multiplied by 10:  $2 \times 3$  tens can be found by simply locating the same basic fact in the tens column.

hundreds	tens	ones
		○○○
		○○○
		$2 \times 3 = 6$

hundreds	tens	ones
	○○○	
	○○○	
	$2 \times 3 \text{ tens} = 6 \text{ tens}$	
	$6 \text{ tens} = 60$	

In the subsequent lesson, place value understanding becomes more abstract as students model place value strategies using the associative property (**3.NBT.3**, **3.OA.5**).  $2 \times 30 = 2 \times (3 \times 10) = (2 \times 3) \times 10$ . The final lesson focuses on solving two-step word problems involving multiples of 10 and equations with unknown quantities (**3.OA.8**). As in the final lesson of Topic E, students estimate to assess the reasonableness of their solutions (**3.OA.8**).

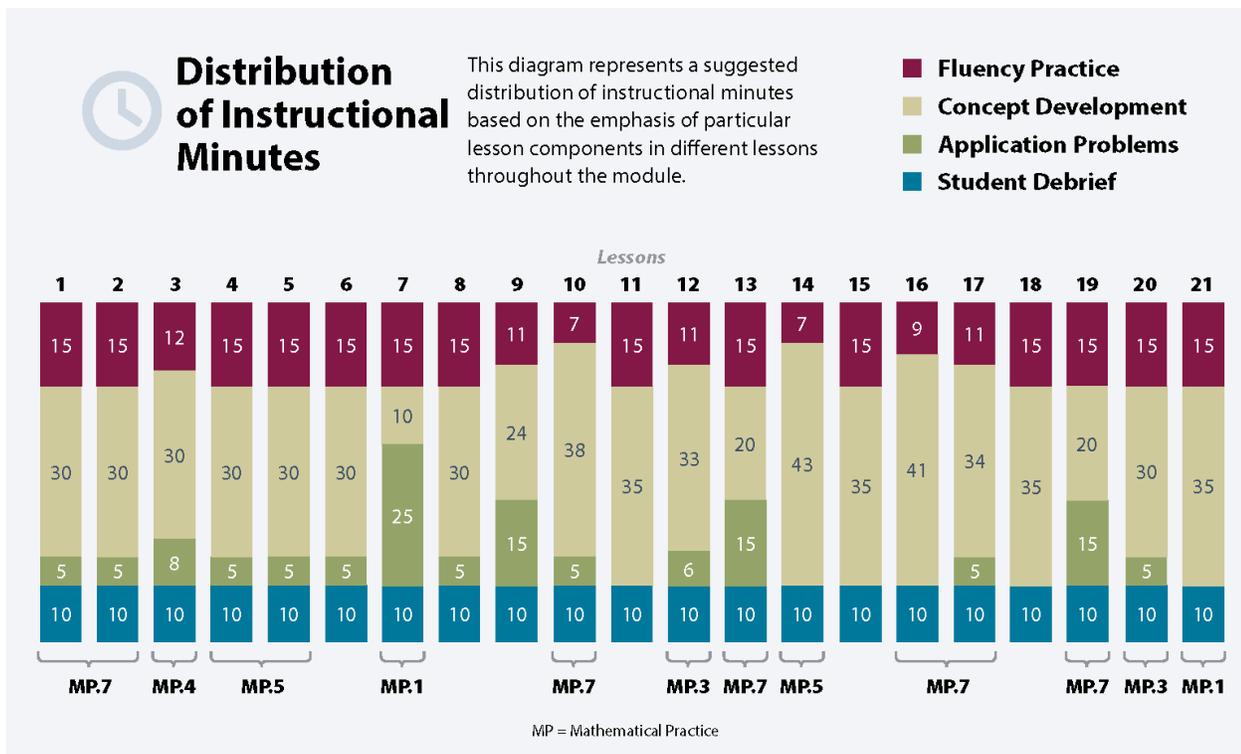
### Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Omit Lessons 6 and 10. Both lessons involve using the distributive property with multiplication and division, a recurring objective in Module 3. Within later distributive property lessons, incorporate units of 6 and 7.

Omit Lesson 11, a problem solving lesson involving multiplication and division. Lesson 11 shares an objective with Lesson 15 and is also similar to Lesson 7.

Omit Lesson 13. Study its essential understandings, and embed them into the delivery of Lesson 14’s Concept Development. Modify Lesson 14 by omitting Part 1 of the Concept Development, a part which relies on the foundation of Lesson 13.



## Focus Grade Level Standards

### Represent and solve problems involving multiplication and division.<sup>1</sup>

- 3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Glossary, Table 2.)
- 3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations  $8 \times ? = 48$ ,  $5 = \_ \div 3$ ,  $6 \times 6 = ?$ .*

### Understand properties of multiplication and the relationship between multiplication and division.<sup>2</sup>

- 3.OA.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If  $6 \times 4 = 24$  is known, then  $4 \times 6 = 24$  is also known. (Commutative property of multiplication.)  $3 \times 5 \times 2$  can be found by  $3 \times 5 = 15$ , then  $15 \times 2 = 30$ , or by  $5 \times 2 = 10$ , then  $3 \times 10 = 30$ . (Associative property of multiplication.) Knowing that  $8 \times 5 = 40$  and  $8 \times 2 = 16$ , one can find  $8 \times 7$  as  $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ . (Distributive property.)*

### Multiply and divide within 100.<sup>3</sup>

- 3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that  $8 \times 5 = 40$ , one knows  $40 \div 5 = 8$ ) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

### Solve problems involving the four operations, and identify and explain patterns in arithmetic.<sup>4</sup>

- 3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

<sup>1</sup>The balance of this cluster is addressed in Module 1.

<sup>2</sup>The balance of this cluster is addressed in Module 1.

<sup>3</sup>From this point forward, fluency practice with multiplication and division facts is part of the students' on-going experience.

<sup>4</sup>After being fully taught in Module 3, this standard (as well as 3.OA.3) continues to be practiced throughout the remainder of the school year.

- 3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

### Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)<sup>5</sup>

- 3.NBT.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g.,  $9 \times 80$ ,  $5 \times 60$ ) using strategies based on place value and properties of operations.

## Foundational Standards

- 2.OA.3** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
- 2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
- 2.NBT.2** Count within 1000; skip-count by 5s, 10s, and 100s.
- 3.OA.1** Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as  $5 \times 7$ .*
- 3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as  $56 \div 8$ .*
- 3.OA.6** Understand division as an unknown-factor problem. *For example, find  $32 \div 8$  by finding the number that makes 32 when multiplied by 8.*

## Focus Standards for Mathematical Practice

- MP.1** **Make sense of problems and persevere in solving them.** Students engage in exploratory lessons to discover and interpret patterns, and they apply their observations to solving multi-step word problems involving all four operations.
- MP.3** **Construct viable arguments and critique the reasoning of others.** As students compare solution strategies, they construct arguments and critique the reasoning of their peers. This practice is particularly exemplified in daily Application Problems and problem-solving specific lessons in which students share and explain their work with one another.
- MP.4** **Model with mathematics.** Students use arrays, tape diagrams, and equations to represent word problem situations.

<sup>5</sup>The balance of this cluster is addressed in Module 2.

**MP.5 Use appropriate tools strategically.** Students analyze problems and select the appropriate tools and pathways to solutions. This is particularly evident as students select problem-solving strategies and use arithmetic properties as simplifying strategies when appropriate.

**MP.7 Look for and make use of structure.** In this module, patterns emerge as tools for problem solving. For example, students make use of structure as they utilize the distributive property to establish the  $9 = 10 - 1$  pattern, or when they check the solution to a fact using units of 9 by making sure the sum of the digits in the product adds up to 9. They make use of the relationship between multiplication and division as they determine unknown factors and interpret their meanings.

## Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives	Days
<b>3.OA.4</b> <b>3.OA.5</b> <b>3.OA.7</b> <b>3.OA.9</b> 3.OA.1 3.OA.2 3.OA.3 3.OA.6	<b>A The Properties of Multiplication and Division</b> Lesson 1: Study commutativity to find known facts of 6, 7, 8, and 9. Lesson 2: Apply the distributive and commutative properties to relate multiplication facts $5 \times n + n$ to $6 \times n$ and $n \times 6$ where $n$ is the size of the unit. Lesson 3: Multiply and divide with familiar facts using a letter to represent the unknown.	3
<b>3.OA.3</b> <b>3.OA.4</b> <b>3.OA.5</b> <b>3.OA.7</b> 3.OA.1 3.OA.2 3.OA.6	<b>B Multiplication and Division Using Units of 6 and 7</b> Lesson 4: Count by units of 6 to multiply and divide using number bonds to decompose. Lesson 5: Count by units of 7 to multiply and divide using number bonds to decompose. Lesson 6: Use the distributive property as a strategy to multiply and divide using units of 6 and 7. Lesson 7: Interpret the unknown in multiplication and division to model and solve problems using units of 6 and 7.	4
<b>3.OA.3</b> <b>3.OA.4</b> <b>3.OA.5</b> <b>3.OA.7</b> 3.OA.1 3.OA.2 3.OA.6 3.OA.8	<b>C Multiplication and Division Using Units up to 8</b> Lesson 8: Understand the function of parentheses and apply to solving problems. Lesson 9: Model the associative property as a strategy to multiply. Lesson 10: Use the distributive property as a strategy to multiply and divide. Lesson 11: Interpret the unknown in multiplication and division to model and solve problems.	4



Standards	Topics and Objectives	Days
	Mid-Module Assessment: Topics A–C (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
<b>3.OA.3</b> <b>3.OA.4</b> <b>3.OA.5</b> <b>3.OA.7</b> <b>3.OA.9</b> 3.OA.1 3.OA.2 3.OA.6	<b>D Multiplication and Division Using Units of 9</b> Lesson 12: Apply the distributive property and the fact $9 = 10 - 1$ as a strategy to multiply. Lessons 13–14: Identify and use arithmetic patterns to multiply. Lesson 15: Interpret the unknown in multiplication and division to model and solve problems.	4
<b>3.OA.3</b> <b>3.OA.7</b> <b>3.OA.8</b> <b>3.OA.9</b> 3.OA.1 3.OA.2 3.OA.4 3.OA.6	<b>E Analysis of Patterns and Problem Solving Including Units of 0 and 1</b> Lesson 16: Reason about and explain arithmetic patterns using units of 0 and 1 as they relate to multiplication and division. Lesson 17: Identify patterns in multiplication and division facts using the multiplication table. Lesson 18: Solve two-step word problems involving all four operations and assess the reasonableness of solutions.	3
<b>3.OA.5</b> <b>3.OA.8</b> <b>3.OA.9</b> <b>3.NBT.3</b> 3.OA.1	<b>F Multiplication of Single-Digit Factors and Multiples of 10</b> Lesson 19: Multiply by multiples of 10 using the place value chart. Lesson 20: Use place value strategies and the associative property $n \times (m \times 10) = (n \times m) \times 10$ (where $n$ and $m$ are less than 10) to multiply by multiples of 10. Lesson 21: Solve two-step word problems involving multiplying single-digit factors and multiples of 10.	3
	End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further application 1 day)	2
<b>Total Number of Instructional Days</b>		<b>25</b>

## Terminology

### New or Recently Introduced Terms

- Multiple (specifically with reference to naming multiples of 9 and 10, e.g., 20, 30, 40, etc.)
- Product (the quantity resulting from multiplying two or more numbers together)

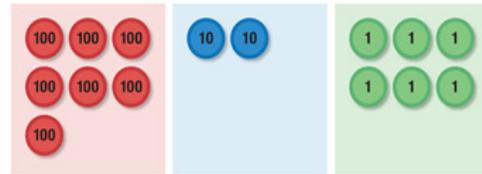
### Familiar Terms and Symbols<sup>6</sup>

- Array (a set of numbers or objects that follow a specific pattern)
- Commutative property (e.g.,  $2 \times 3 = 3 \times 2$ )
- Distribute (with reference to the distributive property; e.g., in  $12 \times 3 = (10 \times 3) + (2 \times 3)$ , the 3 is the multiplier for each part of the decomposition)
- Divide, division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g.,  $15 \div 5 = 3$ )
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group, and the other is a multiplier that indicates the number of groups)
- Equation (a statement that two expressions are equal, e.g.,  $3 \times 4 = 12$ )
- Even number (a whole number whose last digit is 0, 2, 4, 6, or 8)
- Expression (a number, or any combination of sums, differences, products, or divisions of numbers that evaluates to a number, e.g.,  $8 \times 3$ ,  $15 \div 3$ )
- Factors (numbers that are multiplied to obtain a product)
- Multiply, multiplication (an operation showing how many times a number is added to itself, e.g.,  $5 \times 3 = 15$ )
- Number bond (model used to show part–part–whole relationships)
- Number sentence (an equation or inequality for which both expressions are numerical and can be evaluated to a single number, e.g.,  $21 > 7 \times 2$ ,  $5 \div 5 = 1$ )
- Odd number (a number that is not even)
- Ones, twos, threes, etc. (units of one, two, or three)
- Parentheses (the symbols ( ) used around a fact or numbers within an equation, expression, or number sentence)
- Quotient (the answer when one number is divided by another)
- Row, column (in reference to rectangular arrays)
- Tape diagram (a method for modeling problems)
- Unit (one segment of a partitioned tape diagram)
- Unknown (the *missing* factor or quantity in multiplication or division)
- Value (how much)

<sup>6</sup>These are terms and symbols students have used or seen previously.

## Suggested Tools and Representations

- Array
- Number bond (model used to show part–part–whole relationships)
- Place value disks (pictured at right)
- Tape diagram (a method for modeling problems)



## Scaffolds<sup>7</sup>

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units*, please refer to “How to Implement *A Story of Units*.”

## Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic C	Constructed response with rubric	3.OA.3 3.OA.4 3.OA.5 3.OA.7 3.OA.9
End-of-Module Assessment Task	After Topic F	Constructed response and timed fluency with rubric	3.OA.3 3.OA.4 3.OA.5 3.OA.7 3.OA.8 3.OA.9 3.NBT.3

<sup>7</sup>Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website [www.p12.nysed.gov/specialed/aim](http://www.p12.nysed.gov/specialed/aim) for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.